TDC part II ① paper III Group A Optics

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FERMAT'S PRINCIPLE OF LEAST TIME

Statement :- "When a light ray travels between two points P and Q then it follows a path out of all possible paths from P to Q which requires the least time."

Explanation:- Suppose P and Q are two points in the same medium as shown in fig-1. Light takes time dt to travel the distance dL in the medium.

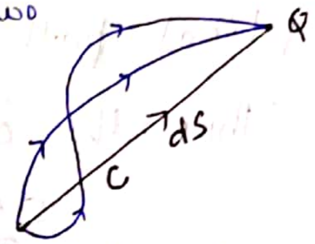

Fig-1

So $$dt = \frac{dL}{v} \quad \text{——— (1)}$$

where v = velocity of light in the medium.

The time taken by Light to travel the total distance PQ in the medium is

$$t = \int_P^Q \frac{dL}{v} \quad \text{——— (2)}$$

$$\Rightarrow t = \int_P^Q \frac{dL}{c/\mu} \qquad \because v = \frac{c}{\mu}.$$

$$\Rightarrow t = \frac{1}{c} \cdot \int_P^Q \mu \cdot dL \quad \text{——— (3)}$$

The quantity $\int_P^Q \mu . dL$ is known as optical path Δ.

If we consider different paths from P to Q then from Fermat's principle of least time,

$$\frac{dt}{ds} = 0 \quad \text{——— (4)}$$

where ds is a parameter that expresses the difference between any two given paths under comparision.

equ (3) may be written as

$$t = \frac{\Delta}{c} \quad \text{——— (5)}$$

From equⁿ ⑤ it is clear that the time of traverse is directly proportional to the optical path length Δ.

Therefore, Fermat's principle can be defined in terms of optical path length Δ as follows.

" Light travels along a path having minimum optical path length Δ".

Thus the condition may now be expressed as

$$\frac{d\Delta}{ds} = 0 \qquad \text{—————— ⑥}$$

FERMAT'S PRINCIPLE OF EXTREMUM PATH

" A light ray travelling from one point to another point will follow a path for which, compared to all neighbouring paths, the time required is either a maximum or minimum or stationary".

It is known as Fermat's principle of extremum path or Fermat's principle of stationary time.

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LAWS OF REFLECTION USING FERMAT'S PRINCIPLE

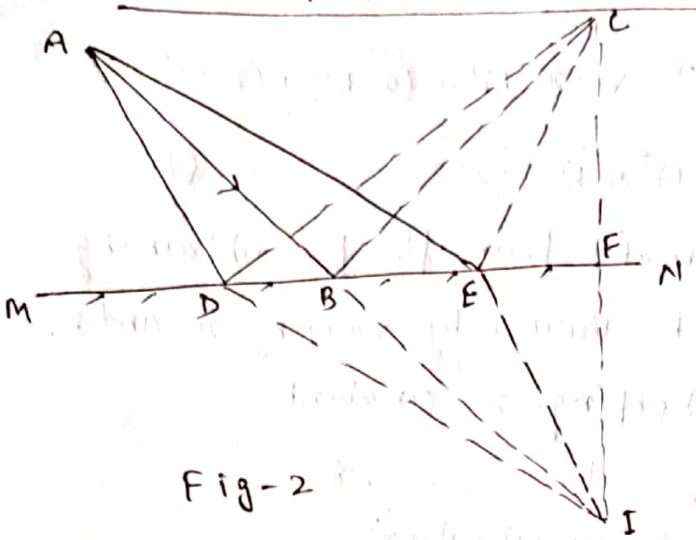

Fig-2

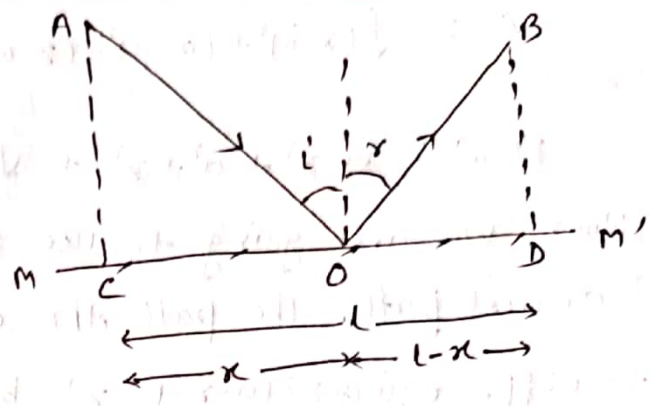

Fig-3.

Suppose a plane mirror $M_1M_2M_3M_4$ lies in X-Z plane. A and B are two points above the plane mirror located in a plane ABCD in XY plane normal to the mirror plane. Light coming from point A is reflected towards the point B as shown in fig-4.

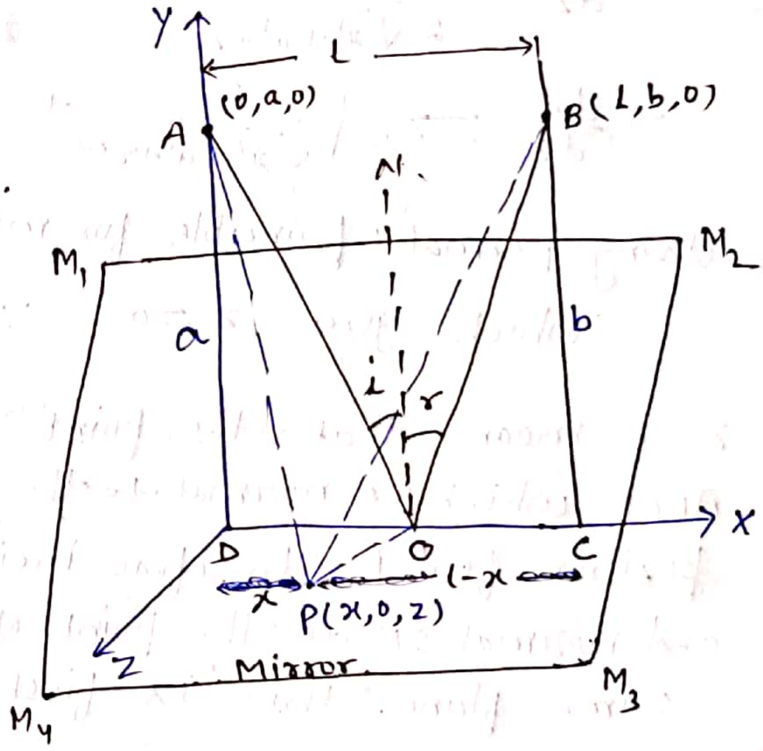

Fig-4

Suppose the light ray passes through a point $P(x,0,z)$ in Z-X plane (Mirror plane). It means that the light is incident along AP and it is reflected along PB. Thus APB is a most general conceivable path from A to B.

Take origin at D. Let DC and DA be the x and y axes respectively. Let $DA = a$, $DC = L$, $BC = b$. The point P has general coordinates $(x, 0, z)$.

If $AP + PB = \Delta$ (optical path length) then we get

$$\Delta = \sqrt{(x-0)^2+(0-a)^2+(z-0)^2} + \sqrt{(x-l)^2+(0-b)^2+(z-0)^2}$$

$$\Rightarrow \Delta = \sqrt{x^2+a^2+z^2} + \sqrt{(x-l)^2+b^2+z^2} \quad \text{——— (7)}$$

Now we are going to use Fermat's principle for obtaining actual path. The path APB can be varied by varying x and z.

Diff. equⁿ (7) w.r.t z by keeping x constant

$$\frac{d\Delta}{dz} = \frac{1}{2\sqrt{x^2+a^2+z^2}}\cdot 2z + \frac{1}{2\sqrt{(x-l)^2+b^2+z^2}}\cdot 2z$$

$$\Rightarrow \frac{d\Delta}{dz} = z\left[\frac{1}{\sqrt{x^2+a^2+z^2}} + \frac{1}{\sqrt{(x-l)^2+b^2+z^2}}\right] \quad \text{——— (8)}$$

Using Fermat's principle, for minimum optical path length, $\frac{d\Delta}{dz} = 0$

Which gives $z = 0$ $\quad \because \frac{1}{\sqrt{x^2+a^2+z^2}} + \frac{1}{\sqrt{(x-l)^2+b^2+z^2}} > 0$

$z=0$ means that the point P must lie in the plane ABCD which is normal to the mirror plane. O is such a position for P. Therefore incident ray OA, reflected ray OB and normal ON at the point of incidence 'O' lie in the same plane. This is first law of reflection.

For second law of reflection, using $z=0$ in equⁿ (7), we get

$$\Delta = \sqrt{x^2+a^2} + \sqrt{(x-l)^2+b^2} \quad \text{——— (9)}$$

Diff equⁿ (8) w.r.t. x, we get

$$\frac{d\Delta}{dx} = \frac{1}{2\sqrt{x^2+a^2}}\cdot 2x + \frac{1}{2\sqrt{(x-l)^2+b^2}}\cdot 2(x-l)$$

$$\Rightarrow \frac{d\Delta}{dx} = \frac{x}{\sqrt{x^2+a^2}} + \frac{x-l}{\sqrt{(x-l)^2+b^2}} \quad \text{——— (10)}$$

Using Fermat's principle, for minimum optical path length, $\frac{d\Delta}{dx} = 0$

$$\Rightarrow \frac{x}{\sqrt{x^2+a^2}} + \frac{x-l}{\sqrt{(x-l)^2+b^2}} = 0 \Rightarrow \frac{x}{\sqrt{x^2+a^2}} = \frac{l-x}{\sqrt{(x-l)^2+b^2}}$$

From fig, $\sin i = \frac{x}{\sqrt{x^2+a^2}}$, $\sin r = \frac{l-x}{\sqrt{(x-l)^2+b^2}}$

$\Rightarrow \sin i = \sin r \Rightarrow i = r$

It is second law of reflection.